1. You run a cafe. You sell coffee at a price of $2.00/cup and sell 250 cups each day. A tea house nearby sells tea at $2.50/cup with sales of 200 cups. A year ago, you lowered the price from $2.50/cup to try to compete, and found that you had an extra 25 customers (25 cups). A month ago, the tea house raised its price from $2.25/cup and lost 25 customers, who switched to buying coffee from your cafe.

(a) Assume and solve for the exponential demand curve.

\[ Q_c = \alpha P_c^\beta t^\gamma \]

\[ \varepsilon_D = \frac{dQ_c}{dP_c} \frac{P_c}{Q_c} = \frac{-25}{0.50} \frac{2.50}{250} = -0.55 \]

\[ \varepsilon_{Q_c,P_t} = \frac{dQ_c}{dP_t} \frac{P_t}{Q_c} = \frac{25}{0.25} \frac{2.25}{225} = 1 \]

\[ Q_c = \alpha P_c^{-0.55} P_t \]

\[ 250 = \alpha (2.00)^{-0.55} 2.50 \]

\[ \alpha = 146 \]

\[ Q_c = 146 P_c^{-0.55} P_t \]

(b) Assume a linear demand function and determine how much you should change your revenue maximizing price when your competitor changes their price.

\[ Q_c = \alpha - \beta P_c + \gamma P_t \]

\[ \beta = \frac{dQ_c}{dP_c} = \frac{-25}{0.50} = -50 \]

\[ \gamma = \frac{dQ_c}{dP_t} = \frac{25}{0.25} = 100 \]

\[ Q_c = \alpha - 50P_c + 100P_t \]

\[ 250 = \alpha - 50(2.00) + 100(2.50) \]

\[ \alpha = 100 \]

\[ Q_c = 100 - 50P_c + 100P_t \]
\[
\begin{align*}
TR &= Q_cP_c = 100P_c - 50P_c^2 + 100P_tP_c \\
\frac{dTR}{dP_c} &= 100 - 100P_c + 100P_t = 0 \\
P^*_c &= 1 + P_t \\
\frac{d^2TR}{dP_t^2} &= -100 < 0 \\
\frac{dP^*_c}{dP_t} &= 1
\end{align*}
\]

2. A garage door company sells to the residential housing market. Income is a critical aspect of this market, because increases in income results in new homes being built (and new garage doors installed) and/or the installation of new garage doors in existing homes. Currently, the company sells 10,000 doors at $1,500 per door. If income increases from $32,000 to $34,000, the company expects to sell 12,000 units. If income stays constant but the company increases its price by $100, they estimate that they will sell 8,500 doors.

Assume that the demand curve is characterized by \( Q_g = \alpha - \beta P_g + \gamma I^{0.5} \). What is the revenue maximizing price at each income level?

Note that \( \Delta I = $2000 \) \( \rightarrow \Delta Q_g = 2000 \) and that \( \Delta P = $100 \) \( \rightarrow \Delta Q_g = -1500 \)

\[
\begin{align*}
Q_g &= \alpha - \beta P_g + \gamma I^{0.5} \\
\beta &= \frac{dQ_g}{dP_g} = \frac{-1500}{100} = -15
\end{align*}
\]

From description, \( \varepsilon_{Q_g,I} = \frac{dQ}{dI} = \frac{2000}{20000 \cdot 10000} = 3.2 \)

From demand func., \( \varepsilon_{Q_g,I} = \frac{dQ}{dI} = 0.5\gamma I^{-0.5} \frac{I}{Q} \\
= 0.5\gamma \frac{I^{0.5}}{Q} \\
So, \varepsilon_{Q_g,I} = \varepsilon_{Q_g,I} \rightarrow \frac{0.5\gamma I^{0.5}}{Q} = 3.2 \\
\frac{0.5\gamma(32000)^{0.5}}{10000} = \gamma = 357.6
\]
\[ Q_g = \alpha - 15P_g + 357.6I^{0.5} \]
\[ 10000 = \alpha - 15(1500) + 357.6(32000)^{0.5} \]
\[ \alpha = -31469 \]
\[ Q_g = -31469 - 15P_g + 357.6I^{0.5} \]
\[ TR = -31469P_g - 15P_g^2 + 357.6I^{0.5}P_g \]
\[ \frac{dTR}{dP_c} = -31469 - 30P_g + 357.6I^{0.5} = 0 \]
\[ P^*_g = -1049 + 11.92I^{0.5} \]
\[ P^*_g(I = \$32000) = \$1083 \]
\[ P^*_g(I = \$34000) = \$1149 \]