ECNS 309, Fall 2014

Practice Problems #4

1. Show that at P1 and P2, both producers and consumers are better off within a market scenario than under a no market scenario. Suppose that the utility function is $U = Q_1^{1/2} Q_2^{1/3}$, the production possibility frontier is $Q_1 = 10 - Q_2^2$, and in the market scenario, prices are $P_1 = 2$ and $P_2 = 1$.

2. For each production function, (i) graph the total, average, and marginal product functions and (ii) identify the three production stages.

   (a) $Q = 0.5x + x^2 - 0.25x^3$

   (b) $Q = 3x - 0.5x^2$

   (c) $Q = \begin{cases} 1.5x, & \text{if } 0 \leq x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$

3. For each production function, determine the input demand function.

   (a) $Q = 2x - x^2$

   (b) $Q = x + 0.2x^2 - x^3$

4. Consider the following production function, output price, and input price combinations. Solve for the profit-maximizing input and output amounts as well as the revenue, variable cost, and profit values. Then, graph the total, average, and marginal value products and the revenues, costs, and profits.

   (a) $Q = 3x - 0.25x^2$, $P = 5$, $w = 2$.

   (b) $Q = 0.5x + x^2 - 0.1x^3$, $P = 3$, $w = 1$.

   (c) $Q = 1 + x - 2x^2$, $P = 4$, $w = 1$.

5. Discuss the mathematical and economic intuition of when and why optimal production strategies change when $Q \neq 0$ when $x = 0$.

6. Determine the degree of homogeneity for the following production functions. Interpret.

   (a) $Q(x, z) = \dfrac{(x-z)^2}{x}$

   (b) $Q(x, z) = \dfrac{z}{(x-z)^2}$

(c) $\text{Yield}(\text{Fertilizer},\text{Rain}) = \dfrac{\text{Fertilizer}^{3/2}}{\text{Rain}}$

7. Provide an economic description for the following production function using the production-side and cost-side approaches. When determining the cost-side approach, determine the homogeneity of the supply, input demand, and profit functions.

   (a) $Q = 3 + x^{0.5}$

   (b) $Q = x^{1/3}/2$ and $FC = 2$

   (c) $Q = (x - 2)^{0.8}$

8. Suppose that the production function is characterized as $Q = \alpha x^{\beta}$, where α and β are some constant parameters.

   (a) Derive the supply, input demand, and indirect profit functions.

   (b) Now suppose that there is a technological improvement such that $\alpha = (\alpha + \theta)$, where θ represents a change in technology. Re-derive the supply, input demand, and indirect profit functions.

   (c) Suppose that $\alpha > 0$ and $0 < \beta < 1$. Discuss the effects on supply, input demand, and profit if (i) $\theta < 0$ and (ii) $\theta > 0$.

9. Consider the following production function: $Q = x_1^{1/2} x_2^{1/3}$.

   (a) Derive the profit-maximizing supply function, input demands, and indirect profit function.

   (b) Determine the homogeneity conditions.

10. Consider the following production function: $Q = x_1^{1/2} x_2^{1/3}$.

    (a) Derive the profit-maximizing supply function, input demands, and indirect profit function.

    (b) Determine the homogeneity conditions.

11. Consider the following production function: $Q = x_1^{3/5} x_2^{1/5}$.

    (a) Derive the profit-maximizing supply function, input demands, and indirect profit function.

    (b) Determine the homogeneity conditions.

12. Consider the general production function: $Q = x_1^{\alpha} x_2^{\beta}$, where α and β are simply constants representing exponents.

    (a) Derive the profit-maximizing supply function, input demands, and indirect profit function.

    (b) Determine the homogeneity conditions.