ECNS 309, Fall 2014

Practice Problems #3—Solutions

1. Estimating the demand function for retail-level gasoline.

   (a) Answers will vary. An example of a model for retail-level gasoline is,

   $$Q_{\text{gas}} = \alpha - \beta_1 P_{\text{gas}} + \beta_2 I - \beta_3(\#\text{ Hybrids}) + \beta_4(\text{Pop})$$

   where I represents median income, (# Hybrids) represents the number of hybrid cars in a location, and (Pop) represents the population.

   (b) Consider the model $Q_{\text{gas}} = \alpha + \beta P_{\text{gas}} + e$.

      i. Regression parameters.

      $$\begin{aligned} \hat{\alpha} &= 10795 \\ \hat{\beta} &= -4439 \end{aligned}$$

      ii. Standard error and hypothesis test.

      $$SE_{\hat{\beta}} = 1378$$

      $$\begin{aligned} H_0 &: \quad \hat{\beta} - 0 = 0 \\ H_a &: \quad \hat{\beta} \neq 0 \\ t_{\text{stat}} &= -3.22 \\ t_{\text{crit (2-tailed},\alpha=0.05,df=8-2)} &= 2.447 \\ |t_{\text{stat}}| > t_{\text{crit}} &: \quad \text{Reject null hypothesis} \end{aligned}$$

      iii. Adjusted R^2 for the regression model.

      $$adj\ R^2 = 0.573$$

      iv. Within the context of your findings in (i)–(iii), discuss the economic interpretation and implications of these findings.

      The model has a relatively good fit to the data ($R^2 = 0.573$) and the parameter of interest, β, is statistically different from zero at a 95% confidence level. The marginal effect, $\frac{dQ}{dP} = -4439$, indicates that, on average in the Rocky Mountain region of the United States, a one dollar deviation is correlated with a -4439 gallon decrease in the consumption of gasoline per day. Because it is very unlikely that gas prices will change by such a large amount, it is more appropriate to interpret the estimated marginal effect in cents. For example, for a 10 cent deviation in the price of gasoline, on average, the daily gas consumption will change by -444 gallons.

(c) Consider the model $Q_{\text{gas}} = \alpha + \beta_1 P_{\text{gas}} + \beta_2 \text{Pop} + e$.

i. Regression parameters.

$$\begin{aligned} \hat{\alpha} &= 7074 \\ \hat{\beta}_1 &= -2975 \\ \hat{\beta}_2 &= 92 \end{aligned}$$

ii. Standard error and hypothesis test.

$$SE_{\hat{\beta}_1} = 957$$

$$\begin{aligned} H_0 : &\quad \hat{\beta}_1 - 0 = 0 \\ H_a : &\quad \hat{\beta}_1 \neq 0 \\ t_{\text{stat}} &= -3.11 \\ t_{\text{crit (2-tailed},\alpha=0.05,df=8-3)} &= 2.571 \\ |t_{\text{stat}}| > t_{\text{crit}} : &\quad \text{Reject null hypothesis} \end{aligned}$$

$$SE_{\hat{\beta}_2} = 28$$

$$\begin{aligned} H_0 : &\quad \hat{\beta}_2 - 0 = 0 \\ H_a : &\quad \hat{\beta}_2 \neq 0 \\ t_{\text{stat}} &= 3.30 \\ t_{\text{crit (2-tailed},\alpha=0.05,df=8-3)} &= 2.571 \\ |t_{\text{stat}}| > t_{\text{crit}} : &\quad \text{Reject null hypothesis} \end{aligned}$$

iii. Adjusted R^2 for the regression model.

$$adj\ R^2 = 0.838$$

iv. Within the context of your findings in (i)–(iii), discuss the economic interpretation and implications of these findings.
The model has an even better fit to the data ($R^2 = 0.838$), implying that including population as a variable in the gasoline demand model is appropriate. The parameters of interest, β_1 and β_2, are both statistically different from zero at a 95% confidence level. The marginal effect, $\frac{dQ}{dP} =$

-2975, indicates that, on average in the Rocky Mountain region of the United States, a one dollar deviation is correlated with a -2975 gallon decrease in the consumption of gasoline per day. The marginal effect, $\frac{dQ}{d\text{Pop}} = 92$, indicates that, on average in the Rocky Mountain region of the United States, a one million person deviation is correlated with a 92 gallon increase in the consumption of gasoline per day.

2. Estimating the exponential demand function, $Q_i = \alpha P_i^{\beta_1} P_j^{\beta_2} e$.

   (a) Parameter estimates.

   First, determine the correlation matrix.

	log Q_i	log P_i	log P_j
log Q_i	1	-0.86	0.56
log P_i	-0.86	1	0.39
log P_j	0.56	0.39	1

   From here, you can estimate the parameters.

$$\hat{A} = 1.42$$
$$\hat{\beta}_1 = -0.79$$
$$\hat{\beta}_2 = 0.87$$

   (b) Discuss the economic interpretation of the estimated parameters.

   The parameters are interpreted as elasticities.

   (c) Given the estimated parameter values, present the demand function in its original exponential form.

$$Q = 4.14 P_i^{-0.79} P_j^{0.87}$$

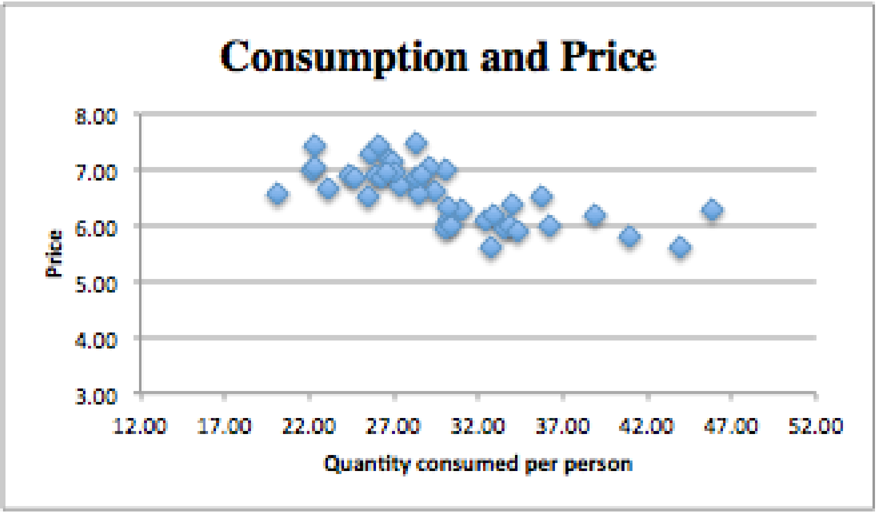
3. You are considering the market for beer. You need to empirically estimate the demand function for this product.

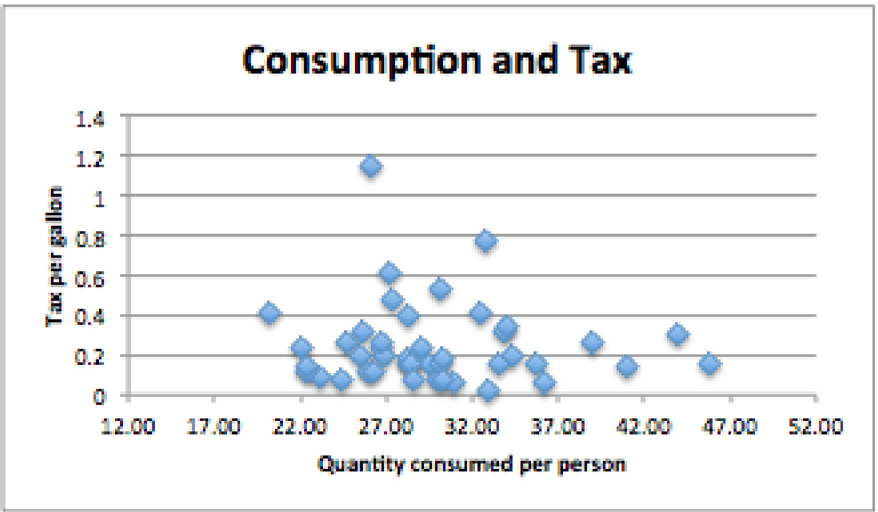
   (a) Consider the per-capita beer demand model, $\frac{\text{Consumption}}{\text{Person}} = \alpha + \beta_1 \text{Price} + \beta_2 \text{Tax} + e$.

      i. Standard summary statistics about the central tendency and dispersion of each variable in the model.

	Per Capita Consumption	**Price**	**Tax**
Mean	29.84	6.58	0.26
Median	28.61	6.62	0.20
Variance	33.97	0.26	0.05
Std Deviation	5.83	0.51	0.21
CV	0.20	0.08	0.83

ii. Graphically represent the relationship between each of the variable pairs.





iii. Estimate the model parameters and interpret.

$$\begin{array}{rcccccc} Q_{\text{beer}} & = & 81.86 & - & 7.86P & - & 0.98(\text{Tax}) \\ & & (8.52) & & (1.29) & & (3.09) \end{array}$$

Performing hypotheses tests indicates that only the estimated parameter associated with the price variable is statistically different from zero at a 95% confidence level. This implies that a one dollar deviation in the price of six units of beer is, on average, correlated with a -7.86 gallon per person effect in beer demand. The marginal effect associated with per gallon taxes on beer is not statistically different from zero, suggesting that there is no statistical relationship between taxes and beer consumption behavior. Economically, this might suggest that consumers are not sensitive to taxation on alcoholic drinks.

iv. Determine and interpret the model fit statistic, R^2.

The adjusted $R^2 = 0.47$. This implies that 47% of the variation in beer demand across the states in the sample is explained by the variables included in the empirical demand model.

(b) Now consider the per-capita exponential beer demand model, $\frac{\text{Consumption}}{\text{Person}} = \alpha \text{Price}^{\beta_1} \text{Tax}^{\beta_2} e$.

i. Estimate the model parameters and interpret.

$$\begin{array}{rcccccc} \ln Q_{\text{beer}} & = & 6.50 & - & 1.67(\ln P) & - & 0.007(\ln \text{Tax}) \\ & & (0.85) & & (0.45) & & (0.05) \end{array}$$

Performing hypotheses tests indicates, as in the linear model, that only the estimated parameter associated with the log-transformed price variable is statistically different from zero at a 95% confidence level. Estimated parameters can be interpreted directly as elasticities. This implies that a 1% deviation in the price of six units of beer is, on average, correlated with a -1.67% gallon per person change in beer demand. This suggests that the own-price elasticity of demand is close to unit-elastic, indicating that producers are pricing their beer products optimally; that is, the price strategies for an average retailer maximizes revenue.

ii. Determine and interpret the model fit statistic, R^2.

The adjusted $R^2 = 0.48$ is slightly higher than in the linear model. This suggests that the log-log transformation may provide a better characterization of the beer demand function.

iii. Use the parameter estimates to represent the estimated beer demand in the original exponential form.

$$Q_{\text{beer}} = 665 P^{-1.67} T^{-0.007}$$