ECNS 309, Fall 2014

Practice Problems #2

1. Rigorously define a "random variable." Describe how statistical tools can be used to characterize random variables.

2. Consider the Poisson distribution. Similar to the Binomial distribution, the Poisson probability distribution function (pdf) characterizes the number of occurrences out of some number of opportunities. The Poisson pdf is characterized as:

$$f(k; v) = \text{Prob}(x = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where k represents the number of times that an event occurs, and $\lambda = p \cdot n$ such that n is the total number of opportunities during which an event can occur and p is the probability that an event occurs.

Suppose that you are operating a food delivery service that typically delivers to 50 locations per day ($n = 50$). Your company guarantees that the delivery will be made in under 30 minutes. You would like to determine the probability that it will take longer than 30 minutes to make a delivery.

   (a) Assume that the probability of at least one late delivery is 0.15. What is the probability of making 5 late deliveries?

   (b) Assume that the probability of at least one late delivery is 0.25. What is the probability of making 3 late deliveries?

3. Suppose that you trying to determine the number of different combinations of name initials.

   (a) If all people have two given names (a first and a middle) and one surname, how many total unique combinations are possible?

   (b) If some people have only one given name and surname and other have two given names, how many total unique combinations of initials are possible?

4. Consider the following statement:

   All roses are flowers.

   Some flowers fade quickly.

   Therefore, some roses fade quickly.

   Does the conclusion follow from the premise? Discuss.

5. Suppose that the following table describes the pdf of stock portfolio returns and of the number of stocks that are in each portfolio.

Return	**Prob[Return]**	**# stocks**	**Prob[# stocks]**
12%	0.02	4	0.2
5%	0.11	7	0.08
8%	0.15	2	0.02
13%	0.05	1	0.01
3%	0.18	10	0.25
7%	0.1	12	0.1
6%	0.09	6	0.04
10%	0.06	5	0.06
4%	0.13	15	0.05
9%	0.08	8	0.05
11%	0.03	9	0.14

Calculate the following:

(a) Expected value of each variable. Interpret the expected value.

(b) Variance of each variable. Interpret the variance.

(c) Standard deviation of each variable. Interpret the standard deviation.

(d) Skewness of each variable. Interpret the skewness.

6. Consider the following table that describes the sales of cars observed across Bozeman's car dealerships and the associated sales prices.

# of sales	**Price**
245	$22,150
165	$26,850
205	$19,200
188	$28,500
104	$28,750
195	$20,000
204	$21,450
277	$18,000
145	$23,150
190	$20,050
245	$15,800
196	$21,900
98	$32,145
115	$27,110

The following have to be done by hand. You can use Excel to check your answers, but it is important that you understand how to solve these statistics without the aid of a computer.

(a) Determine the mean of each variable.

(b) Determine the variance of each variable.

(c) Determine the standard deviation of each variable.

(d) Determine the coefficient of variation of each variable.

(e) Set up and test the hypothesis that the average number of sales in the Bozeman area is greater than 210 cars at the 5% significance level (95% confidence level).

(f) Set up and test the hypothesis that the average price is equal to $21,000 at the 5% significance level (95% confidence level).

(g) Determine the covariance between the quantity and price variables. Interpret the resulting measure.

(h) Determine the correlation between the quantity and price variables. Test whether the correlation statistic is significantly different than zero. Interpret the resulting measure.

(i) Suppose that you would like to estimate the linear demand function, $Q_{\text{sales}} = \alpha + \beta P$. What are the estimates of $\hat{\alpha}$ and $\hat{\beta}$.

(j) Interpret the $\hat{\beta}$ coefficient.

7. Suppose that $x = y$.

   (a) Calculate the covariance, S_{yx}.

   (b) Calculate the correlation, ρ_{yx}.

   (c) Now suppose that $x = \alpha y$. Calculate the correlation ρ_{yx}.

8. You are considering opening up a coffeeshop in Montana. Knowing that to-go coffee is often considered a luxury good, you would like to determine whether there is a relationship between the number of coffeeshops in a Montana county and the median income in that county. You will need to use either Excel or another statistical package to complete this exercise.

   (a) Download the data set **ps2_data.xls** from the course website (`http://www.montana.edu/bekkerman/classes/ecns309/ps2_data.xls`). The data set contains the number of coffeeshops in a county and the associated median income in 2011 (collected from the U.S. Census Bureau).

   (b) Determine the central tendency (mean, median) and the variability (variance, standard deviation, coefficient of variation) statistics for each variable. Discuss these statistics.

(c) Determine the covariance and correlation of each variable. Determine whether the correlation statistic is significantly different than zero. Discuss the results.

(d) Specify a linear function that will explain the relationship between coffeeshops and income in Montana counties. This function should be very general, with unknown parameter values.

(e) Use regression analysis to estimate the parameter values of the function in part (d). Present these results.

(f) Determine whether each parameter estimate is statistically different than zero.

(g) Discuss what we might expect to occur on average to the number of coffeeshops in a county if the median income in a county increased by $1,000.

(h) Using the regression results and the mean values for coffeeshops and income, calculate the demand elasticity of coffeeshops with respect to a change in median income. Discuss the economic insights that can be gleaned from this calculation.

(i) Where should you build a coffeeshop?