ECNS 309

Practice Problems #1—Solutions

1. Evaluate and (if possible) simplify the following:

$$a) \qquad \sum_{i=1}^{10} i = 55$$

$$b) \qquad \sum_{i=1}^{10} 2 = 20$$

$$c) \qquad \sum_{i=0}^{3} 2 \cdot Y^i = 2(1 + Y + Y^2 + Y^3)$$

$$d) \qquad \sum_{i=1}^{5} Y \cdot Y^i = Y^2(1 + Y + Y^2 + Y^3 + Y^4)$$

2. Consider the function $y = 10x^{1/2} - x$. Find the extreme value and determine whether it is a maximum or minimum.

   $x_* = 25$

   $\frac{d^2y}{dx^2} = -2.5x^{-1.5} < 0$. So, the value is a maximum

3. Consider the function $x + y = 12$. Determine what values of x and y will maximize the product $x \times y$.

   $x_* = 6$

   $\frac{d^2y}{dx^2} = -2 < 0$. So, the value of x is a maximum

   $y_* = 6$

4. Suppose that a minor league hockey arena has a maximum capacity of 1,000 seats. The number of tickets sold, Q, is a function of the ticket price, P. The demand function for tickets is $Q = 1000 - 50P$. Find the price and quantity that maximizes the <u>revenue</u> from ticket sales.

   $P_* = \$10$

   $\frac{d^2y}{dx^2} = -100 < 0$. So, this price maximizes total revenue

   $Q_* = 500$

5. Suppose that you are running a small business. Sometimes, you need to run errands in your car. However, you dislike driving and estimate that the cost of "misery" of driving is \$40/hour. In addition, you need to buy gasoline, which costs \$4/gallon. The gas mileage on your car is $MPG = s - 0.015s^2$, where s represents the speed in miles per hour and MPG is the miles per gallon of gas. How fast should you drive when you run the errands to minimize the joint cost of "misery" and gas?

$$s_* = \frac{0.33 \pm \sqrt{0.33^2 - 4(0.00225)(11)}}{2(0.00225)} = (51.22,\ 95.44)$$

$$\frac{d^2C}{ds^2} = \frac{40D}{s^3} + \frac{10D(1-0.03s)^2}{(s-0.015s^2)^3} + \frac{0.12D}{s-0.015s^2)^2}$$

At $s_* = 51.22$, $\frac{d^2C}{ds^2} > 0$ So, total costs are minimized at this speed

At $s_* = 95.44$, $\frac{d^2C}{ds^2} < 0$ So, total costs are maximized at this speed

6. You run an advertising agency that sends out a monthly circular with advertisements for local businesses. To advertise, a business must pay \$3.00 per month to your agency. At this price, you have 500 business that use your agency. From past experience, you know that for every \$0.01 increase in the price of advertising, you will lose one business. For example, if you increase the price to \$3.25/month, you will lose 25 businesses. What fee must you charge to maximize your monthly income?

$P_* = \$4.00$

$\frac{d^2y}{dx^2} = -200 < 0$. So, this price maximizes total revenue

$Q_* = 400$

7. You run a taxi cab company and you get some proportion of the total fare that a taxi cab receives from a ride. Assume that the taxi cab driver earns $1 + m^a$ per ride, where m is the number of minutes per ride. That is, taxi cab drivers charge an initial fee, 1, plus a set amount per mile. On longer rides, drivers usually travel faster (higher speeds) to make more money per minute (thus, $a > 1$). What length of ride would the taxi cab need in order to maximize their fare (and you maximize your revenues)?

$$m_* = \left(\frac{1}{a-1}\right)^{1/a}$$

$\frac{d^2TR}{dm^2} = T(\frac{2}{m^3} + (a-1)(a-2)m^{a-3}) > 0$. So, this ride lenght minimizes total revenue

Since m_* minimizes total revenue, the ride length that maximizes total revenue is either $m = 0$ or $m \to \infty$. That is, either very short rides or very long ones.

8. Consider the following demand function: $Q = 100 - 2P^2$. For what range of prices does this demand function satisfy the Law of Demand? Graph only the feasible portion of this demand curve.

$$P \in [0, 7.07]$$

9. Consider the following demand function: $Q = 100 - 2P + 0.001P^2$. For what range of prices does this demand function satisfy the Law of Demand?

   The maximum quantity is at $P = 0$. There are two values of P at which $Q = 0$— $P = 1948.68$ and $P = 51.32$. At $P = 1948.68$, $\frac{dQ}{dP} > 0$, so we know that the slope is upward sloping and does not satisfy the Law of Demand. At $P = 51.32$, $\frac{dQ}{dP} < 0$. Therefore, the feasible region is $P \in [0, 51.32]$.

10. Consider the following demand function: $Q = 100 - 2P + 0.1P^2$. For what range of prices does this demand function satisfy the Law of Demand? Will we ever observe zero demand? Why or why not?

    Law of demand is satisfied when $P \in [0, 10]$. We will never see a price at which $Q = 0$.

11. Suppose you're the manager for a minor league baseball team. A ticket to one of your team's games is \$10. At this price, you draw an average of 1,000 spectators. The demand elasticity for minor league baseball games is –4.5%. You are considering raising the price by \$0.50.

    (a) If you were to raise the price, determine the number of spectators under the assumption that the demand function is linear. Is it optimal to raise the price?

    At $P = \$10.50$, $Q = 775$. Should not raise the price.

    (b) If you were to raise the price, determine the number of spectators under the assumption that the demand function is exponential. Is it optimal to raise the price?

    At $P = \$10.50$, $Q = 802$. Should not raise the price.

    (c) Find the price and quantity at which total revenue is maximized, under the assumption of a linear demand function.

    The price that maximizes total revenue is $P_* = \$6.11$.

12. Assume that an economy consists of only one product. All income is spent on that product.

    (a) Specify the demand curve for the product.

    The demand curve is $Q = I/P$, where I represents income.

    (b) Derive the own-price demand elasticity.

    The own-price elasticity of demand is $\varepsilon_D = -1$.

    (c) Derive the income elasticity. Discuss why this finding makes sense in the context of the described single-product economy.

    The income elasticity of demand is $\varepsilon_I = 1$. Because all income is spent on a single good, a 1% increase in the income will result in a proportional increase in the consumption of the good.

13. Consider the demand function $Q_D = (5 - 0.5P)(0.2I^{0.5})$ and the supply function $Q_S = 10 + P$. Derive the comparative statics results for changes in income, I. That is, solve for the elasticity of the equilibrium price with respect to income, $\varepsilon_{P,I}$, and the elasticity of the equilibrium quantity with respect to income, $\varepsilon_{Q,I}$. Find the numerical values at $I = 144$ and discuss.

$$\varepsilon_{P,I} = \frac{I^{0.5}}{(0.1I^{0.5}+1)(I^{0.5}-10)}$$
$$\varepsilon_{Q,I} = \frac{0.5}{(0.1I^{0.5}+1)}$$

    At $I = 144$, $\varepsilon_{P,I} = 2.72$ and $\varepsilon_{Q,I} = 0.27\%$.

14. Consider the following. The demand for personal computers is characterized by an own-price elasticity of $\varepsilon_D = -5$, a cross-price elasticity with software of $\varepsilon_{Q_C,P_S} = -4$, and an income elasticity of $\varepsilon_{D,I} = 2.5$. Respond to the following as true, false, or uncertain and justify your answer.

    (a) A price reduction for personal computers will increase both the number of units demanded and the total revenue for the sellers.

    True. When operating on the elastic portion of the demand curve, a price reduction increase sales by proportionally more than the decrease in price, resulting in higher total revenues.

    (b) The cross-price elasticity indicates that a 5% reduction in the price of personal computers will cause a 20% increase in the demand for software.

False. A 5% reduction in the price of *software* will result in a 20% increase in the demand for *personal computers.*

(c) Falling software prices will increase revenues received by sellers of both computers and software.

Ceteris paribus, a reduction in software prices would increase personal computer sales at the existing price and would increase total revenues for computers. It is uncertain, however, what falling software prices will do to the total revenue of software because we do not know the own price elasticity of software demand.

(d) A 2% price reduction would be necessary to overcome the effects of a 1% decline in income.

False. A 1% price reduction would be necessary to offset a 2% decline in income.